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Parallel and Perpendicular Lines, Using Slope

Objectives

- Students will be able to identify if lines are parallel, perpendicular or neither, using slope.
- Students will recognize that perpendicular lines have slopes that are opposite reciprocals of each other.
- Students will be able to produce a parallel and perpendicular line, given an existing graph/equation and point.
- Students will use the parallelism or perpendicularity of lines and segments to solve problems (NCSCOS 2.02)

Common Core State Standards

- Algebra-
    - Creating equations that describe numbers or relationships.
        - Create equations in two or more variables to represent relationships between quantities; graph equations on coordinate axes with labels and scales.
- Geometry-
    - Use coordinates to prove simple geometric theorems algebraically.
        - Prove the slope criteria for parallel and perpendicular lines and use them to solve geometric problems (e.g., find the equation of a line parallel or perpendicular to a given line that passes through a given point).

Materials

- Students should have:
    - Graph Paper
    - Pencils
    - Note Paper
    - Premade Transparencies (Pictures provided, Transparencies are not)

Focus and Review (15 minutes)

Students already know how to find the slope of a line from 2 points. They know point-slope form and also slope-intercept form. The following graphs should be shown via pre-constructed transparencies.
Either the teacher may create the transparency composed of all 5 graphs, or 5 separate transparencies. The separate transparencies are preferable as they may be overlaid to show relationships later in the focus and review. However, the individual transparencies require that they are switched out as students work. This decision should be made by each individual teacher, as different classes and teaching styles call for different methods.
The students should be prompted to find the equation of the line, in slope intercept form, for all of the graphs.

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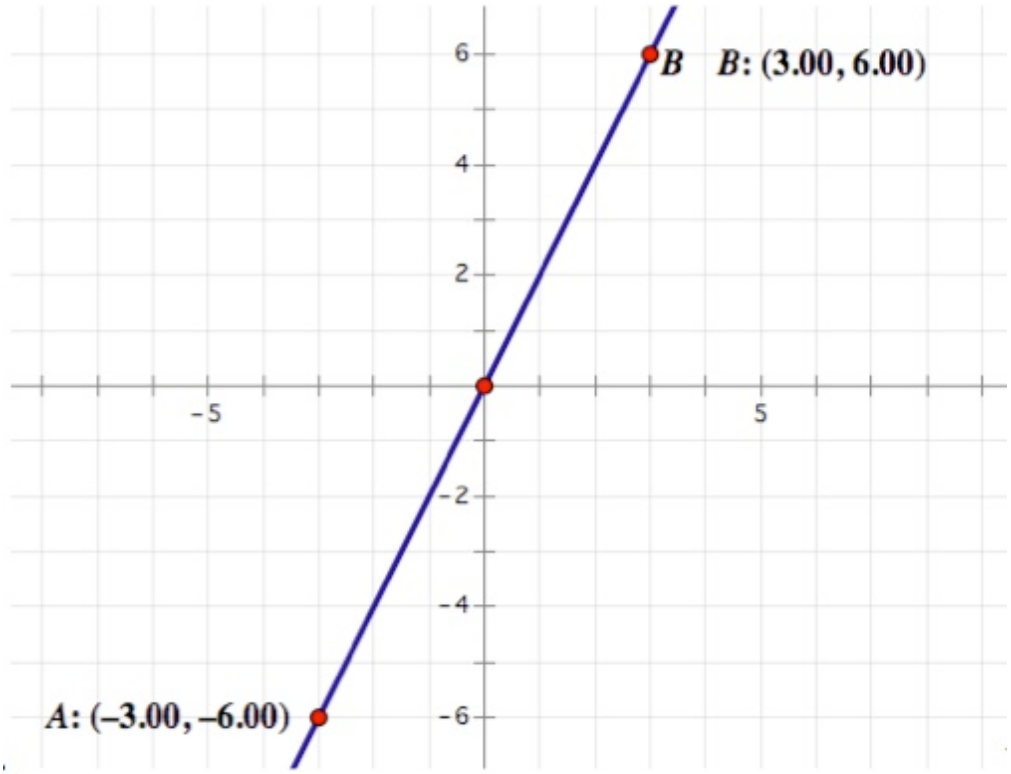

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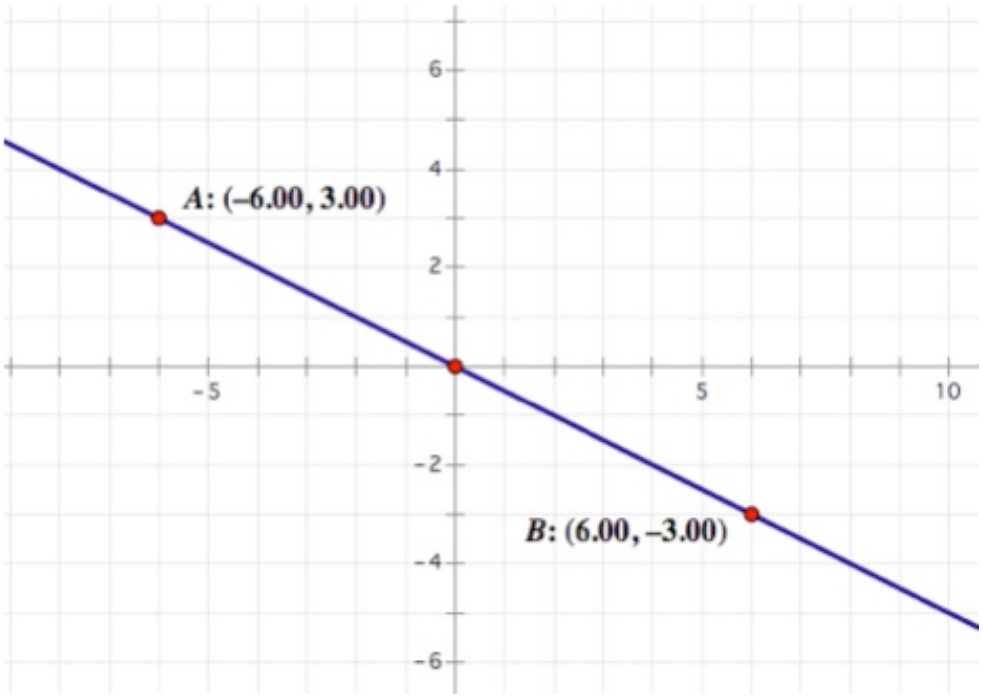

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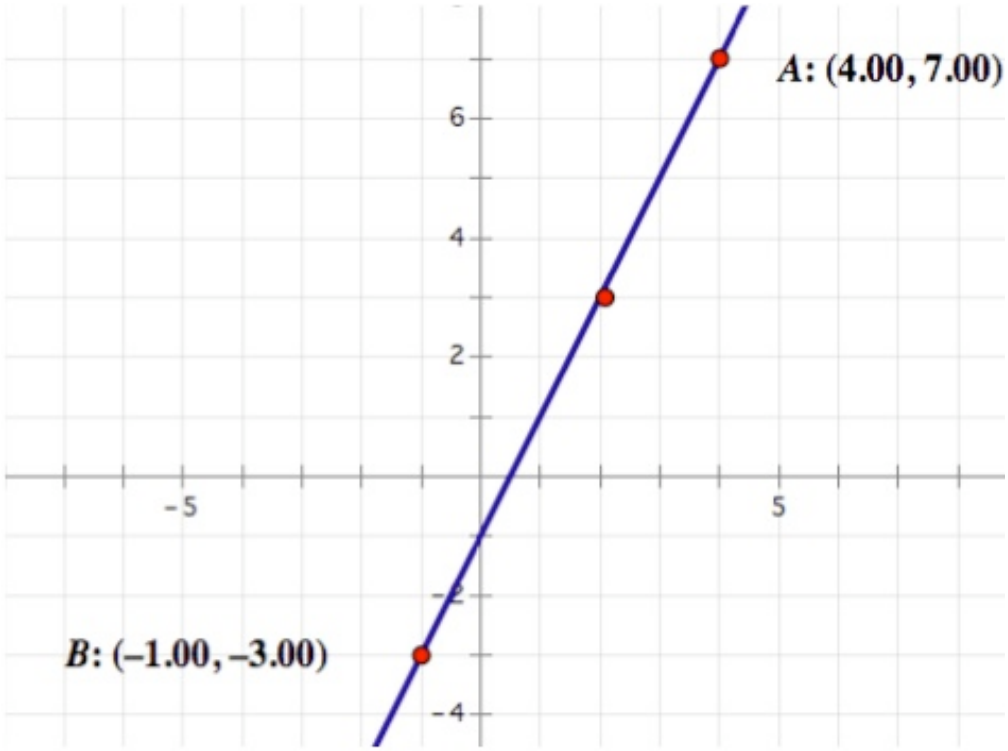

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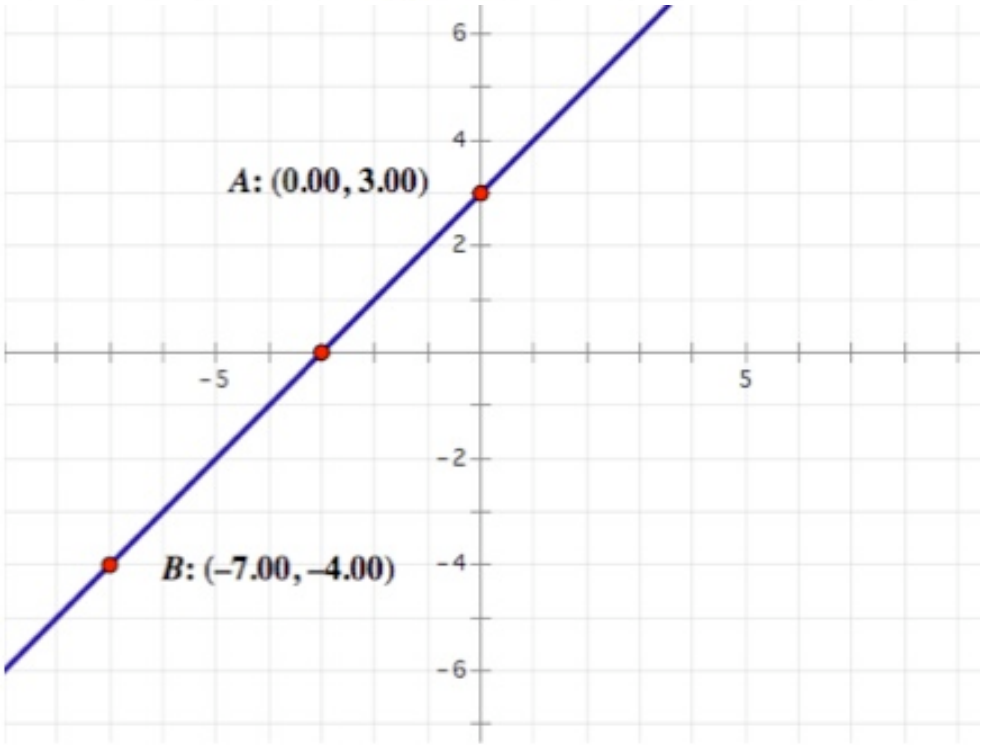

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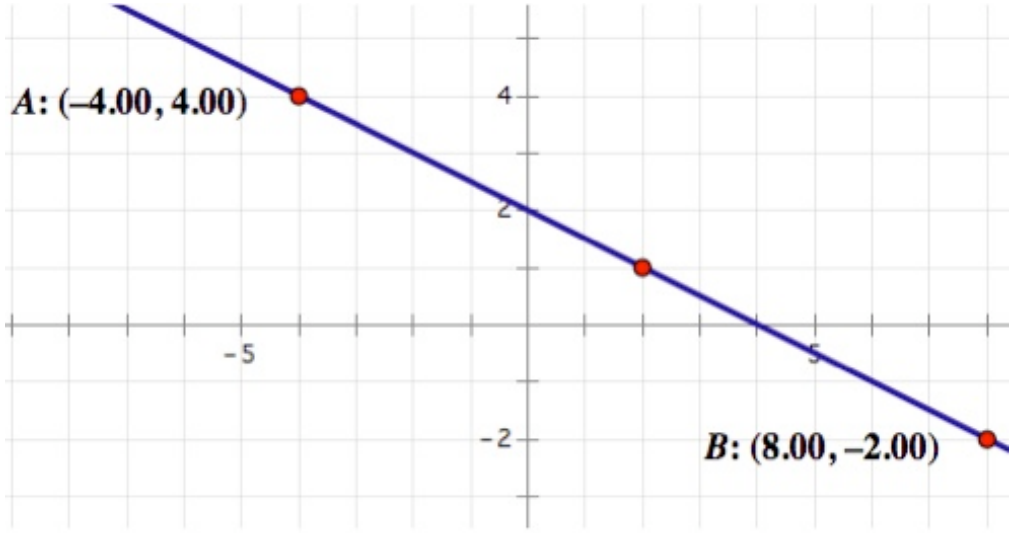


Equations for the provided graphs:

1. **y=2x**
2. **y=-(1/2)x**
3. **y=2x-1**
4. **y=x+3**
5. **y=-(1/2)x+2**

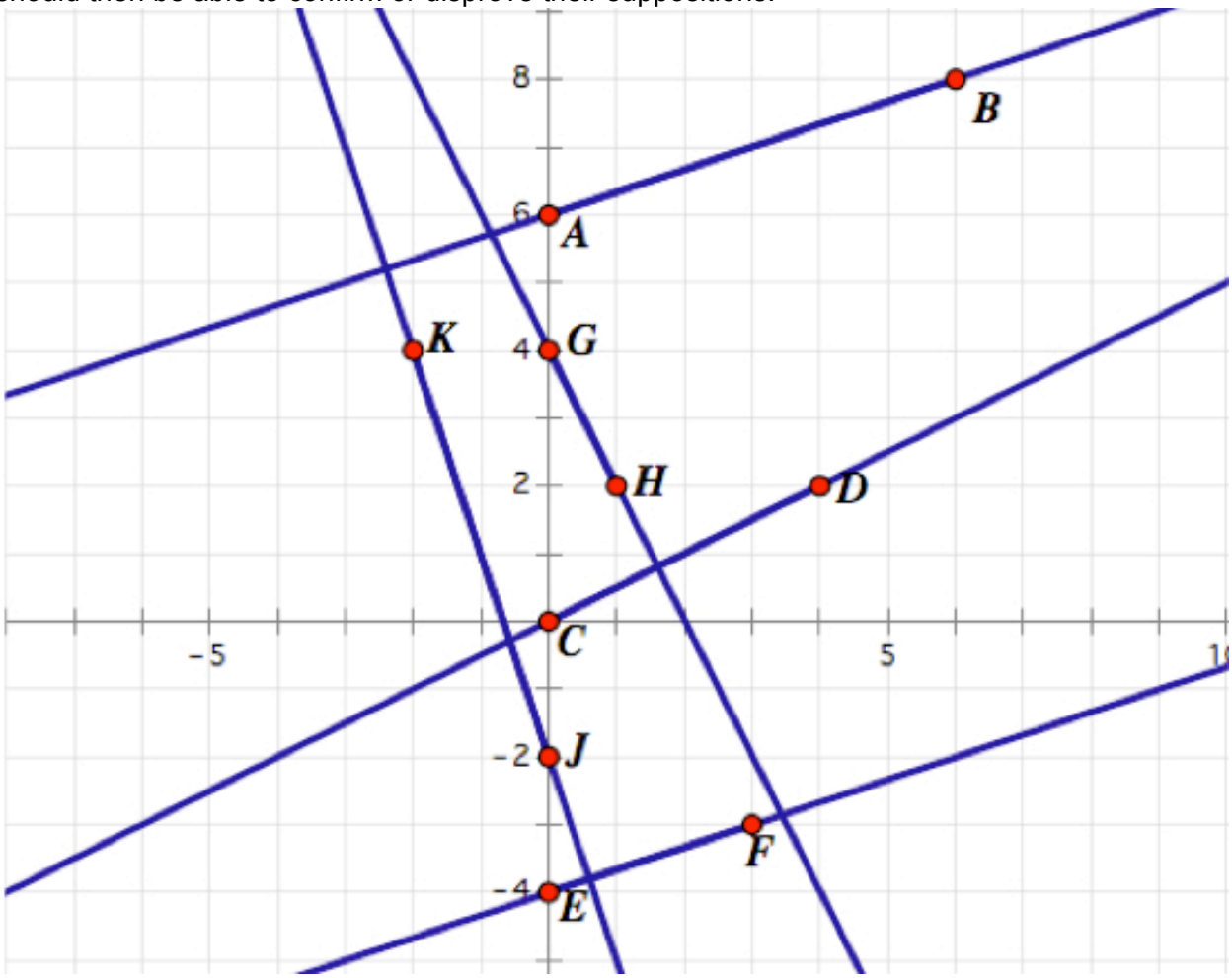
After they have been given a few minutes to calculate for themselves, the class should discuss their findings. *When you put these equations in slope-intercept form, what do you find their slope to be?*

Teacher Input (30 minutes)
Discuss and compare the graphs and slopes of the lines given in the Focus & Review.
Teacher: "*Now what would happen if we put the graphs of 1 and 2 together on the same grid? And what if we added more similar equations to that same grid like y=2x+2 and y=2x+3?*"
If the Teacher has created separate transparencies they may be overlaid in order to show this. Otherwise, the teacher may draw the lines again on the white board or on a transparency.
Students: "The lines have the same slope, but are either moving up or down on the y-axis."
Teacher: "*Nice observation! Now where do these lines intersect?*"
Students: "They don't."
Teacher: "*Never*?"
Students: "No because they are all increasing at the same rate (same slope)."
Teacher: "*Exactly! What are lines called that do not intersect?*"
Students: "**Parallel Lines"**
The teacher writes this on the board, including the definition, while speaking.
Teacher: "*So, now you've found that parallel lines have the same slope, just like in the example we just did where the slope of every line we drew was 2. Now what would it look like if we added the third example to this graph*?"
The teacher adds the other transparency or draws the other line.
Students: "This line forms right angles with the ones it intersects."
Teacher (in a German accent): "*THAT'S A BINGO! And what are lines called that form 90 degree angles with one another*?"
Students: "**Perpendicular Lines"**

The teacher writes this on the board as well as the definition.
Teacher: "*The graph of an equation with the slope -1/2 is perpendicular to the graph of equations with the slope 2. What do you get when you multiply those two slopes, -1/2 and 2*?"
Students: "-1"
Teacher: "*Exactamundo! We can tell if two lines are perpendicular to one another because when we multiply their slopes together, we get -1 in every case. This is because the slopes are opposite reciprocals. So, say that I have a line and I knew the slope was 3. How would I figure out how to form an equation of a line with a perpendicular slope?*"
Students: "Find the opposite reciprocal thingy and use that as our slope?"
Teacher: "*And how! There is one method that works pretty well. If we know 3 times the perpendicular slope is equal to -1 then we can say 3x=-1 and solve for x. So, x= -1/3. Now, let's do some constructions! Show me how to form a line that is parallel to y=(1/4)x + 1 and show us your algebraic reasoning*."
Students can give any variation of y= (1/4)x+n, where n is any integer.
The teacher should ask for several different answers to show how there can be infinity solutions to this type of problem.
Teacher: "*Now I'm going to give you the point (2,-1) and I want you to find THE ONLY line that contains this point AND is parallel to y=(1/4)x+1*."
The teacher helps students by using graph paper with transparency/doc camera to represent this equation and point.
Students: "We know the slope of this new equation must equal the slope of the line it's parallel to, but we don't know the y-intercept to finish the problem."
Teacher: "*So if we use the slope-intercept form, what does our equation look like so far?*"
Students:"y=(1/4)x+b, where b is the unknown y-intercept."
Teacher: "*So, now we have have the outline of this new equation: we have the slope and we also have a point that the line contains. How do we find the y-intercept?"*
Students: "If we plug in the coordinate to the equation, we can solve for b. So this makes b= -1.5"
Teacher: "*FANTASTICO!! Now let's change things up a bit. While using the same original equation y=(1/4)x+1 and the same point (2,-1) show us how to find the ONLY equation that perpendicularly intersects this equation AND contains this point*."
The teacher helps students by using graph paper with transparency/doc camera to represent this equation and point.
Students: "We know the slope of the new equation must be -4 because it's the opposite reciprocal of ¼."
Teacher: "*How did you find that?*"
Students: "I know that ¼ x = -1, so I solved for x to find the perpendicular slope."
Teacher: "*Bravo! Now using the slope-intercept form yet again, what does our equation look like thus far*?"
Students: "y=-4x+b. If we use the same method from before we plug in the same coordinate."
Teacher: "*Why can you use the same coordinate*?"
Students: "Because the new line contains that point, so we can plug it into the equation of the line to find the y-intercept. By doing this we get (-1)=(-4)(2)+b, so b=7. The final equation is y=-4x+7."
Teacher: "*Very good..." *nods wisely, eyes narrowed* "Very good...*"

Guided Practice(25 minutes)
Students will be shown the picture given below, either by transparency or projection. It is composed of 5 lines, with one pair of parallel lines and three pairs of perpendiculars. Students should be asked to guess which lines are parallel and perpendicular BEFORE they begin anything else. After they have finished making their suppositions, they will find the equation for

each line. These equations should be written in slope intercept form. Using what they have been shown about the relationships of slopes between parallel and perpendicular lines, they should then be able to confirm or disprove their suppositions.



The following equations are not given to students. They are only given the graph above. If they need clarification on where the points are placed, that information is listed below.

AB -> y=(1/3)x + 6 {A(0,6),B(6,8)}
CD -> y=(1/2)x {C(0,0),D(4,2)}
EF -> y=(1/3)x - 4 {E(0,-4),F(3,-3)}
GH -> y= -2x + 4 {G(0,4),H(1,2)}
JK -> y= -3x - 2 {J(0,-2),K(-2,4)}

Parallel Pair: AB||EF
Perpendicular Pairs: JK_|_AB, JK_|_EF, GH_|_CD

Possible student responses and mix-ups:

- Students may believe that AB||CD, CD||EF, and GH||JK. This is an assumption that should not be outwardly disputed until they have finished calculating slopes and comparing said slopes. Then the difference in slopes should be brought to their attention. They should be reminded that simply looking at the picture is not enough to determine if lines are parallel.

- Students may believe that GH_|_AB, GH_|_EF, and JK_|_CD. Again, this assumption should not be disputed until they have finished calculating slopes and comparing said slopes. They should be prompted to multiply the slopes to see if the product is -1. Again, simply looking at the picture is not enough to determine if lines are perpendicular.

After the class has discussed the equations found for each line, as well as the perpendicular and parallel pairs, they will move into practice with constructions. These constructions use the graph pictured above (5-Lines Graph).

The teacher should prompt the students with the following problems:

1. Construct a line that passes through **(2, 4)** and is parallel to line CD.
2. Construct a line that passes through **(-3,1)** and is parallel to line GH.
3. Construct a line that passes through **(5, -1)** and is perpendicular to line EF.
4. Construct a line that passes through **(-1, -2)** and is perpendicular to line JK.

Answers for the above questions:

1. $y = (1/2)x + 4$
2. $y = -2x - 5$
3. $y = -3x + 13$
4. $y = (1/3)x - (5/3)$

Possible student mix-ups:

- Students may have difficulty with using the correct line to work from, as in using AB rather than CD. As they may not have seen lines labelled in this manner, it should be explained that using two points that lie upon the line as a line's label is the correct method. However, if they still have difficulties, simply label the lines as L, M, N, P, and Q respectively.
- Many students may incorrectly find the opposite reciprocal when looking for the perpendicular line's slope. They should be gently reminded of the process outlined in the teacher input section.

Independent Practice (10 minutes, finish rest at home)

As students finish, they may begin working on the attached worksheet (labeled "Having FUN with SLOPES!). If they do not finish the worksheet they may complete it for homework.

Answers to Worksheet

1. Parallel
2. Neither
3. Perpendicular
4. Neither - Same Line
5. Perpendicular
6. Parallel
7. $y = (4/5)x - 2$
8. $y = -2x - 1$
9. $y = (3/2)x - 5$
10. $y = 3x + 3$

11. y = -(1/4)x + 1
12. y = -(2/3)x - 3

Closure (10 minutes)
Teacher: "*What did you learn today*?"
Call on students as they raise their hands...
Possible answers:

- "I learned that parallel lines have equal slopes."
- "I learned that perpendicular lines have opposite reciprocal slopes."
- "I learned that a set of 2 lines do not have to be parallel or perpendicular. They can be neither."
- "I learned that you can't be sure 2 lines are parallel just by looking at them...you need to make sure they have the same slope. This is because two lines may not intersect in the window that you are looking at, but they may outside of that window."
- "I learned that slope just means the rate at which a line increases along the x and y axes."

Teacher: "*Good, I think you guys really achieved our objectives today. For tomorrow, make sure you complete your homework and come to class with your thinking caps!*"