Chapter 6.6 Lesson Plan

**Objectives**

* Students will be able to determine whether two lines are parallel or perpendicular based on their slopes.
* Given a line and a point, students will be able to write equations of lines through the point that are parallel or perpendicular to the equation.
* Students will be able to apply properties of parallel and perpendicular lines to geometric representations.

**Common Core State Standards: High School Algebra**

* Creating Equations: Create equations that describe numbers or relationships.
* Seeing Structure in Expression: Interpret parts of an expression, such as terms, factors, and coefficients.

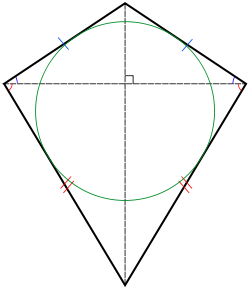
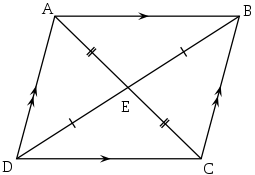
**Materials**

* Computers for each student or group of students
* <http://www.mathwarehouse.com/algebra/linear_equation/interactive-slope-two-lines.php> - Slope of Parallel and Perpendicular Lines Applet
* Graphing calculators for each student
* Time allotted: 90-minute class period

**Four-Phase Lesson Plan**

* **Phase I: Problem Posing**

**Ask students:** “How would you classify the following quadrilaterals?” *kite, parallelogram*

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**Ask students:** “What can you say about the two slats (diagonals) that support a kite?” *They meet at right angles.*

**Tell Students:** “Great! Lines that intersect at right angles are called perpendicular lines. Thus, the diagonals of a kite are perpendicular to one another.”

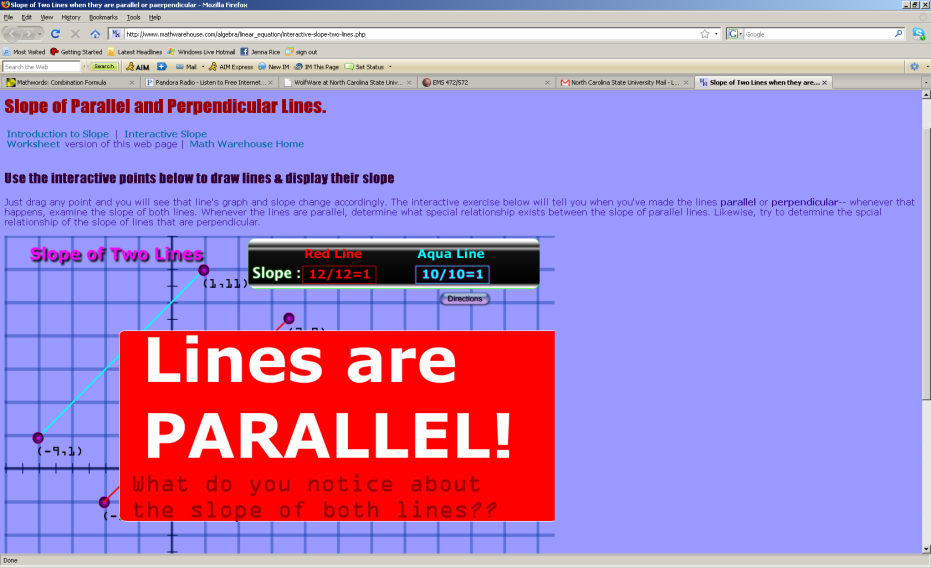
**Ask Students:** “A parallelogram is a quadrilateral in which opposite sides are parallel. By looking at this parallelogram, what do you think it means for two lines to be parallel?”

**Ask students:** “Lines in the same plane that never intersect are called parallel lines. But how do we know mathematically that two lines are parallel or perpendicular?

**Tell students:** Let’s hop on the computer and take a look at the following applet to do some investigating.”

**Phase II: Small Group Investigation**

Students will use the applet listed above to explore the slopes of parallel and perpendicular lines. They will be given a worksheet with questions on it designed to guide their investigation. A screenshot of what the applet looks like is given below. (The red box goes away after a few seconds, and only appears when lines are parallel or perpendicular).



**Worksheet**:

1. Try to make the two lines you see parallel (it will let you know when it is parallel). Write down the equations of the two lines.
2. Repeat the process three more times. What do you notice? Can you make a conjecture about parallel lines based on these observations?
3. Now try to make the two lines perpendicular (it will let you know when they are perpendicular). Write down the equations of the two lines.
4. Repeat the process three more times. What do you notice? Can you make a conjecture about perpendicular lines based on these observations?
5. Based on your observations from the previous problems, which of the following lines are parallel or perpendicular to each other?

a. g.

b. h.

c. i.

d. j.

e. k.

f. l.

**Answer Key:**

Parallel lines—b and l, d and y, h and f

Perpendicular lines—a and e, c and i, k and d, k and g

Neither—j

**Teacher Input:**

After students have completed the worksheet, the teacher will have a class discussion with students.

**Teacher**: *What did you notice about the equations of parallel lines?*

**Student1**: *The slopes of the lines were always the same.*

**Teacher**: *Right! Did you notice anything about the equations of perpendicular lines?*

**Student2**: *The slopes were negative reciprocals. Like if the slope of one line was 4, then the perpendicular line would have a slope of -1/4.*

**Teacher**: *Exactly right. What could we do if we have the equation of one line, and we want to find an equation of a line parallel to it? For example, let’s say we have the line y=4x + 2. What are some equations of lines that would be parallel to it?*

**Student3**: *You could have y=4x, or y=4x-1, and obviously some more lines as long as they have the same slope.*

**Teacher**: *Good job. Now, what could we do if we have an equation of a line, and we want to make a line that is parallel or perpendicular to that line that goes through a specific point? Let’s look at the following example.*

Write an equation in slope-intercept form of the line that passes through (4, 0) and is parallel to the graph of 4x – 3y = 2.

**Teacher**: *Does anyone have any suggestions as to what we should do first?*

**Student1**: *Well we need to know the slope for the new equation, so we should put it in slope-intercept form.*

**Teacher**: *Would someone like to come up to the board and do that?*

Student2 volunteers and puts the following work on the board:

*4x – 3y = 2 🡪 -3y = -4x + 2 🡪 y = (4/3)x – (2/3)*

**Teacher**: *Class, what is the slope of the line for this equation?*

**Class**: *4/3.*

**Teacher**: *What should the slope of our parallel line be then?*

**Student3**: *It should be 4/3 also.*

**Teacher**: *Now we have the slope of our line, and a point on that line, but we still need to find an equation. Which form should we use?*

**Student4**: *Well point-slope form would probably be easiest.*

**Teacher**: *Right. You can also find the equation using slope intercept form. I’ll show both ways.*

Teacher begins working on the board.

*y – y1 = m(x – x1) 🡪 y – 0 = (4/3)\*(x – 4) 🡪 y = (4/3)x – (16/3)*

*You can also solve it in slope-intercept form.*

*y = mx + b 🡪 0 = (4/3)\*4 + b 🡪 0 = (16/3) + b 🡪 b = (-16/3) 🡪 y = (4/3)x – (16/3)*

The teacher then graphs her equations on an overhead calculator that students can see, and she makes sure that the lines are in fact parallel and contain the original point. The teacher is modeling how students should check their work. She also shows them how to check to make sure the original point is on the second line.

**Directions for students**: *Press the “y=” button on the upper left portion of your calculator. Type in the original equation for Y1 and the equation we found for Y2. Then press the “Graph” button on the upper right hand side. Do the lines look parallel? Now, let’s make sure the second line contains our original point. Since we are looking for a specific point, I don’t want to waste time scrolling through to find our x value. Press the “2nd” button, and then “Window” so that you are actually looking at the tblset. Where it says “Indpnt”, make sure “Ask” is highlighted instead of “Auto.” This means that it will ask you for an x value, and when you put one in it will give you the y value. When you have x values far away from zero, this feature is very helpful. Once you have done that, go to “2nd” and “Graph” so you are looking at the table view. For the x column, put the number 4 in since that is the value for our point. Under Y2 we see the corresponding value is zero, so that means that the point (4,0) is on our line.*

After the demonstration, the teacher gives a similar problem but with perpendicular lines for students to try. The teacher should walk around the room and monitor students’ work.

Write an equation in slope-intercept form of the line that passes through the point and is perpendicular to the graph of

**Answer Key:**

First find slope by rewriting the equation in slope-intercept form.

The slope of the line is . The slope of the line perpendicular to this line is the negative reciprocal of , or .

Use the point-slope form to find the equation.

An equation for the line is .

The teacher should ask a student to volunteer his/her answer and graph his/her equations on the calculator. The student should be able to show that his/her equation is right.

After this demonstration, the students are given the second part of the worksheet.

**Worksheet Part 2:**

Write an equation in slope-intercept form of the line that passes through the given point and is parallel to the graph of the given line. Show your work.

1. (9, -3), 5x – 6y = 2
2. (0, 4), 2y = 5x – 7

Write an equation in slope-intercept form of the line that passes through the given point and is perpendicular to the graph of the given line. Show your work.

1. (8, 5), 7x + 4y = 23
2. (0, 0), 9y = 3 – 5x

**Answer Key:**

1. y = (5/6)x – 10.5
2. y = (5/2)x + 4
3. y = (7/4)x – 9
4. y = (9/5)x

**Responses to Activities:**

Students should recognize that parallel lines have the same slope, and the slopes of perpendicular lines are negative reciprocals. Students should know the vocabulary word “reciprocal” by now, but if they don’t, their responses may vary for that question. On the last problem, students should group together lines that are parallel or perpendicular based on the slope, but since some of the lines are not in slope-intercept form, some students may make careless errors that cause their answers to be wrong. Students should understand how to use point-slope and slope-intercept form to find equations that pass through a point and are parallel or perpendicular to a given line. Students should also be able to use the calculator to check the accuracy of their work

* **Phase III: Whole-Class Discussion of Investigation**

Engage students in a whole class discussion about general properties of parallel and

perpendicular lines, as well as special cases. Possible questions to discuss include:

**Ask students: What if we were given a vertical and a horizontal line?**  *They can still be perpendicular however the negative reciprocal rule does not apply in this case.*

**Ask students: If you are given an equation of a line, must you also be given a point in order to find another line that is either parallel or perpendicular to that given equation?** *No you do not have to have a point. Any line that has the same slope (for parallel lines) or the negative reciprocal (for perpendicular lines) will either be parallel or perpendicular to that given equation.*

**Ask students: Can you tell by looking at the graphs of two lines alone if two lines are parallel or perpendicular? Why or why not?** *No you cannot. Sometimes a graph can be deceiving; you should look at the slopes of lines to confirm that the lines are either parallel or perpendicular.*

**Ask students: Can the rules we just learned be extended to more than just two lines? How many lines are parallel or perpendicular to a given line?”** *Yes, it should not matter if there are more than two lines involved. Hopefully students will know that you can find an infinite number of lines that are parallel or perpendicular to a certain line using families of graphs.*

* **Phase IV: Summarizing and Extending**

**Summary:**

To summarize the lesson and bring to a close, ask students:

**“How can we tell if two lines are parallel?”** *Two lines are parallel if their slopes are the same.*

**“How can we tell if two lines are perpendicular?”** *The lines will be perpendicular if their slopes are negative reciprocals of each other.*

**“Describe in your own words what it means for two numbers to be negative reciprocals of each other.”** *Hopefully students will say something like two numbers are negative reciprocals of each other if when they are multiplied together the answer is negative one. They may also say that you could take one number, flip it, and take the opposite sign and the resulting number would be the negative reciprocal.*

**“Does the negative reciprocal rule work for every case”** *No, it does not. It does* ***not*** *work when one line is vertical and one line is horizontal.*

**“Where should we begin when trying to find an equation of a line that is parallel to a given equation through a given point”** *We first have to make sure that the given line is in slope- intercept form.*

**“What would be the next step?”**  *Next, you could plug in the given point along with the appropriate slope in the slope- intercept from of the equation of a line.*

**“How would this process be different if we wanted the line to be perpendicular?”** *The only difference would be that you would have to use a different slope. Instead of the same slope one should the negative reciprocal.*

**Extension Ideas:**

1. To extend the lesson, students could be given a kite and a parallelogram whose vertices are plotted on a coordinate plane and ask them to verify their properties as discussed in the problem posting portion of the lesson. This will allow students to extend the properties of slope into a geometry classroom.
2. Also, students could use a camera and take photos around the school of lines they consider to be parallel or perpendicular. Ask the students to record estimations of the slopes for the lines in these photos. The students could then upload the photos and use a line of best fit to find the slopes of the lines. The students would then compare their estimations to that of the line of best fit and see if indeed the lines were parallel or perpendicular. You could then ask the student to research how the rules for slope could be used in real world construction.