**Materials Needed:**

* Graphing Calculator (2 students per calculator)
* Activity Worksheets

**Rationale**:

Students may never have to plot an exponential function once they leave high school, but every person, no matter the occupation, will have to learn to interpret and understand data and graphs. Throughout this lesson, students will have the chance to gain an understanding of the natural log and exponential functions so that later in life, if a set of data presents itself, the students will be able to notice that the a natural log or exponential function could describe the data and from these functions make conjectures about the data and how the data should act.

**Objectives**:

NCTM

Algebra

* Understanding paterns, relations, and functions
* Expectations: In grades 9-12 all students should—   
  + generalize patterns using explicitly defined and recursively defined functions;
  + understand relations and functions and select, convert flexibly among, and use various representations for them;
  + analyze functions of one variable by investigating rates of change, intercepts, zeros, asymptotes, and local and global behavior;
  + understand and perform transformations such as arithmetically combining, composing, and inverting commonly used functions, using technology to perform such operations on more-complicated symbolic expressions;
  + understand and compare the properties of classes of functions, including exponential, polynomial, rational, logarithmic, and periodic functions;
  + interpret representations of functions of two variables

NCSCOS

Pre-Calculus

Competency Goal 2

**2.01** Use functions (polynomial, power, rational, exponential, logarithmic, logistic, piecewise-defined, and greatest integer) to model and solve problems; justify results.

1. Solve using graphs and algebraic properties.
2. Interpret the constants, coefficients, and bases in the context of the problem.

**2.03** For sets of data, create and use calculator-generated models of linear, polynomial, exponential, trigonometric, power, logistic, and logarithmic functions.

1. Interpret the constants, coefficients, and bases in the context of the data.
2. Check models for goodness-of-fit; use the most appropriate model to draw conclusions or make predictions.

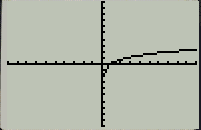
Core Standards

Interpreting Functions

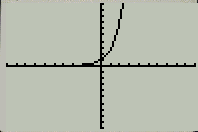
1. Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases.★
   1. Graph exponential and logarithmic functions, showing intercepts and end behavior, and trigonometric functions, showing period, midline, and amplitude.

Building Functions

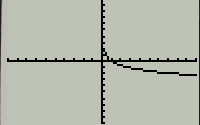
1. (+) Understand the inverse relationship between exponents and logarithms and use this relationship to solve problems involving logarithms and exponents.

**Stage 0 (10 minutes)**

The natural log function ( is a function that, given a value (your independent variable aka x value), will tell you to what value *e* must be raised to give that x value. For example, when x = 1, y = 0 because. The domain for this function is the reals>0 and the range is the reals.

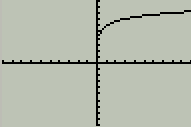


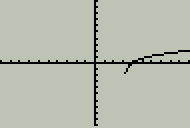
The exponential function is very closely related to the natural log function. This function () takes the independent variable and raises *e* to the independent variable to give you your dependant variable value. For example, when, because. The domain of this function is the reals and the range of this function is the reals>0.

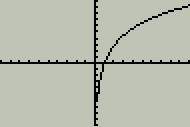


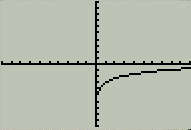
The general form of the natural log function is and the general form of the exponential function is where a, b, c, d, f, and g are constants. Each one of these constants has a different effect on the graph of the functions. Throughout this activity, you will discover what effect each constant has.

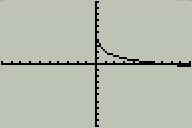
**Stage 1:** (30 minutes) Replicate each graph on your calculator by manipulating either the natural log or the exponential function. For each plot write down what equations you tried to get your graph to match the graph provided. Put a box around your final equation. (I put the answers in red)

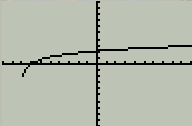


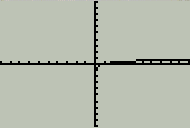


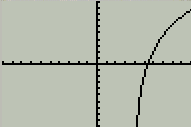


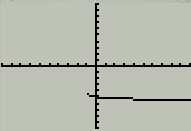


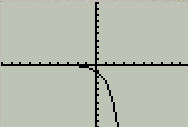


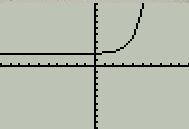


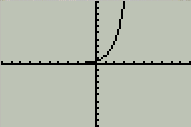


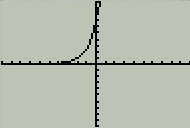


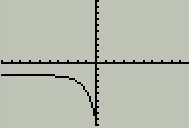


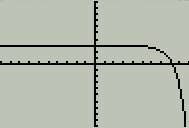


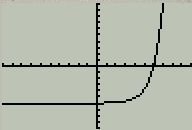












**Stage 2 (10 minutes)**

1. When figuring out what equation produced the graph, what did you look for?

These answers could vary greatly, but I hope they look for the concavity and location of asymptotes.

1. Which of the graphs look similar? Why do you think they look similar?

Kind of like question 1, but I am more interesting learning how they interpreted it.

1. Are the equations you came up with for the similar graphs also similar? Explain why you think the equations are or are not similar.
2. How did you decide whether to use a natural log function or an exponential function?

I see students having the most trouble with this question. The answer is that natural log will never have x-values near negative infinity and the exponential will never have x-values near positive infinity. This may be hard to pick out from this lesson, but I would be sure to bring it up.

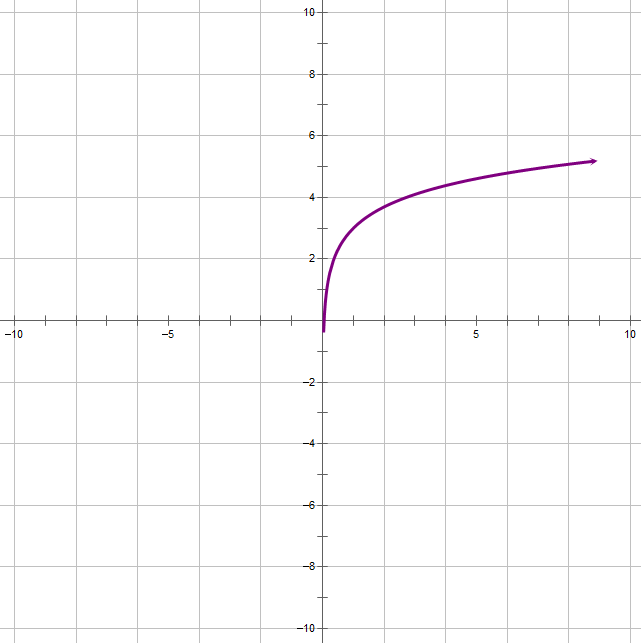
**Stage 3 (25 minutes)**

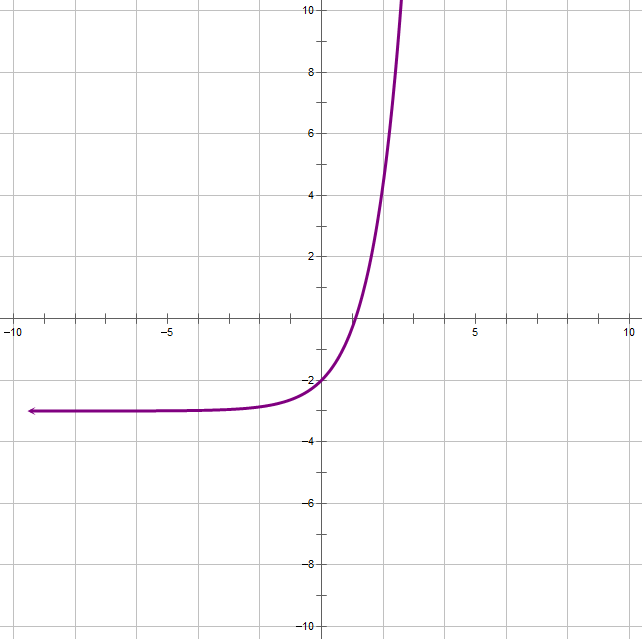
* 1. Earlier it was stated that the general form of the natural log function is and the general form of the exponential function is where a, b, c, d, f, and g are constants. Based on your experience in stage 1, what effect do you think each of the variables has on the graph of the equation and why do you think this? I could see students getting the (x-a) mixed up and instead doing x+a for the standard form (the horizontal shift), which is an understandable mistake.

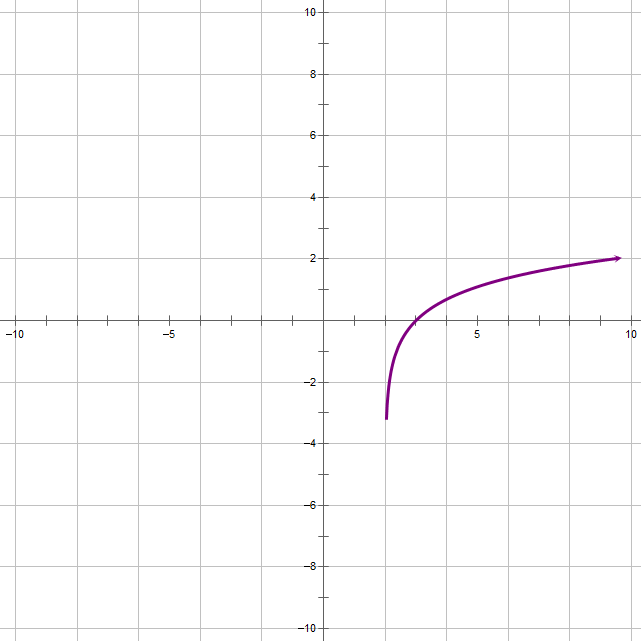


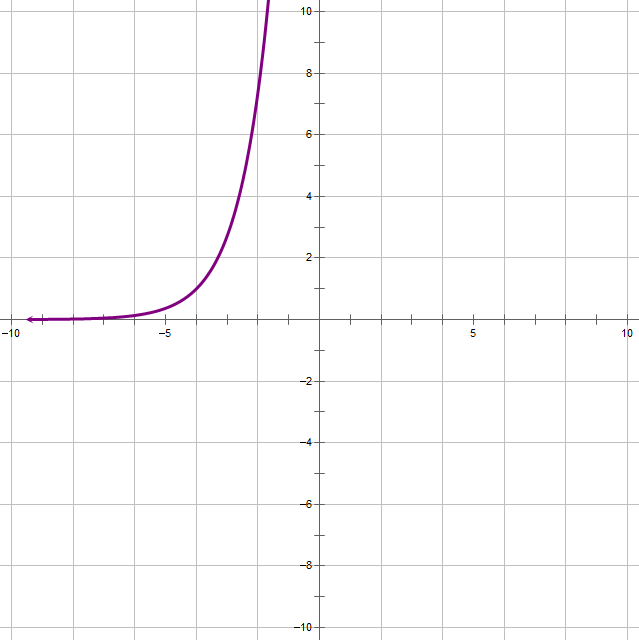


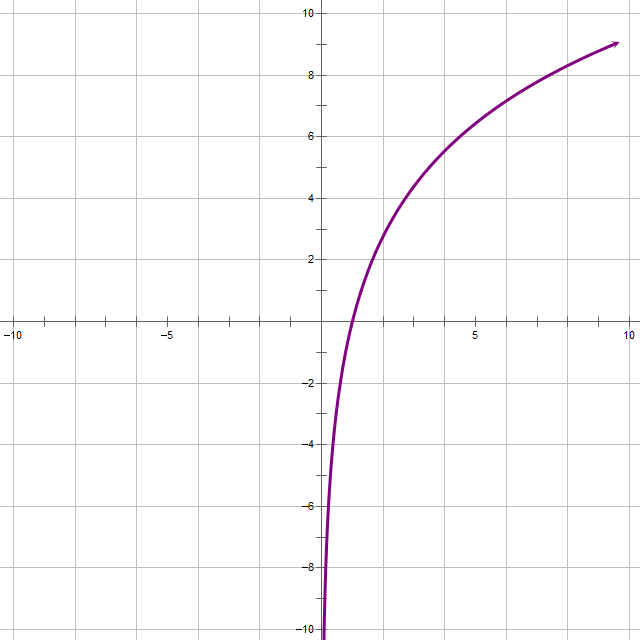

1. Based on your understanding of the effects coefficients on equations and graphs, draw what you expect these graphs to look (on the left coordinate plane) like without graphing them on your calculators. Then graph them on the calculator to check your work. Sketch this graph on the left coordinate plane. (The actual graphs are provided) There is no telling what students could have issues with on these. I would be sure to be walking around and asking question and trying to guide the students in the right direction.

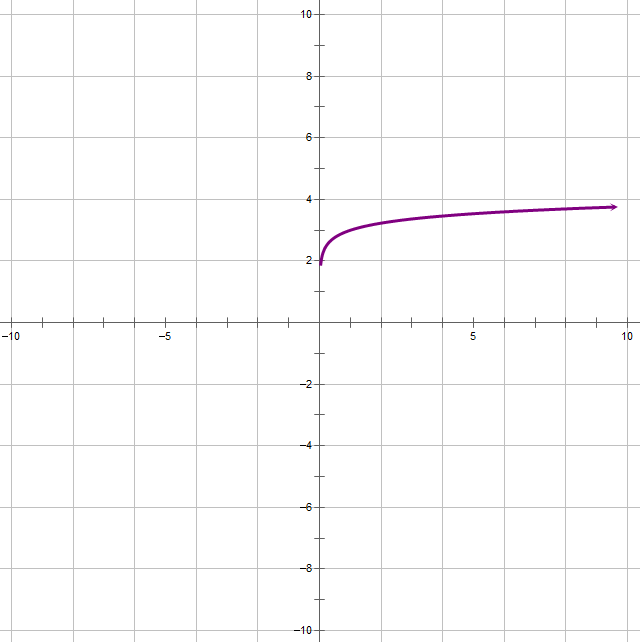
**Actual Graph** 

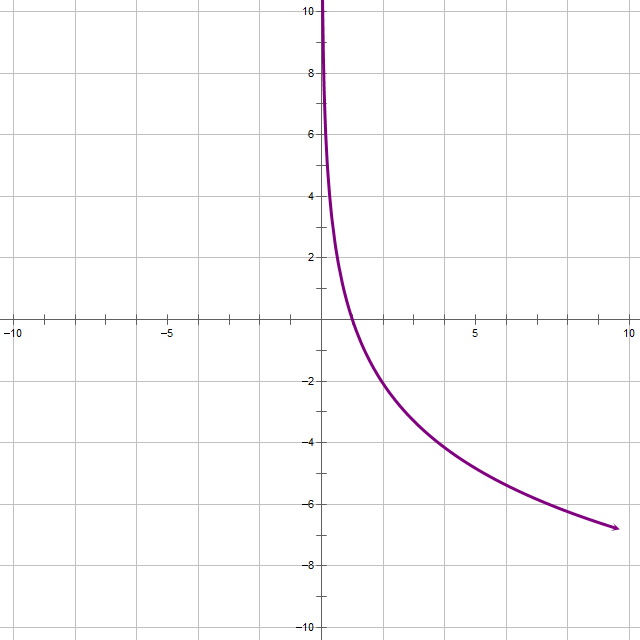
**Actual Graph** 

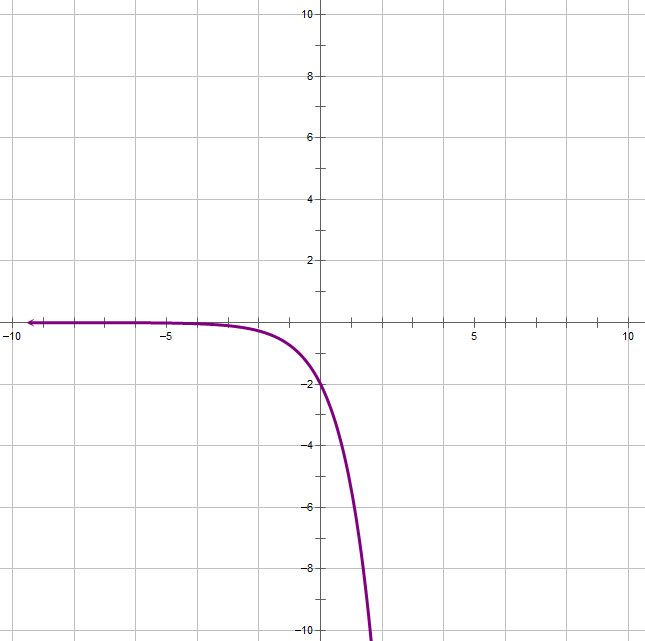
**Actual Graph** 

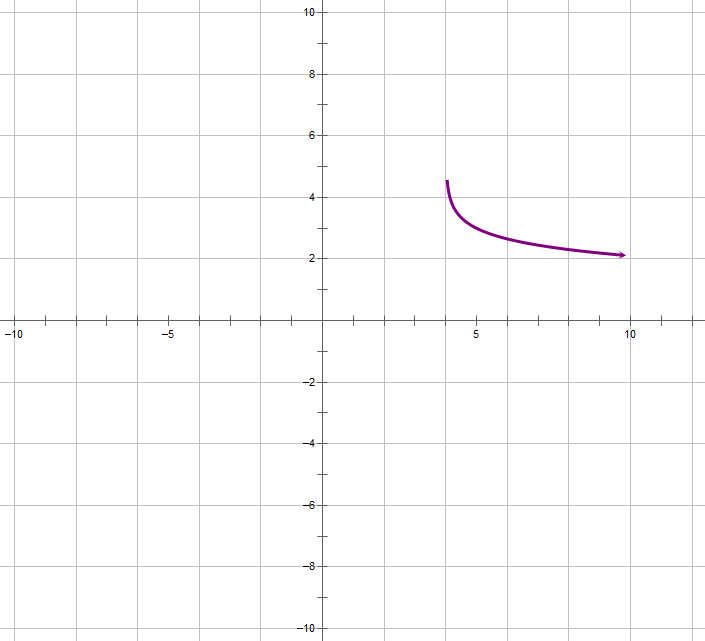
**Actual Graph** 

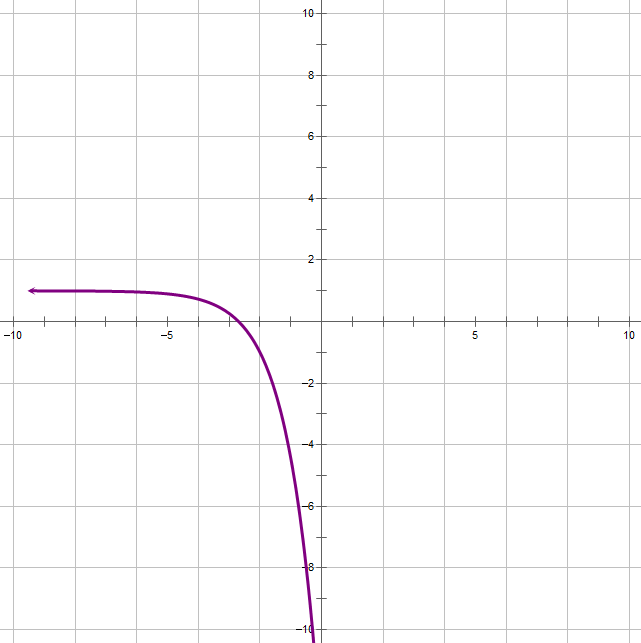
**Actual Graph** 

**Actual Graph** 

**Actual Graph** 

**Actual Graph** 

**Actual Graph** 

**Actual Graph** 

1. Look back at question number 1. Do the variables seem to have the same effect you initially thought? If not, explain what effect you now think they have.

**Stage 4 (15 minutes)**

For this stage, I would lead a group discussion on the natural log and exponential functions. I would ask each pair of students what they thought a variable did to a graph. I would have students fill in this chart on the board. I would also give a copy of this chart to the students for them to fill in and keep for later reference. I expect students would not quite know how to explain the vertical stretch and compression. I consider this to also be closure.

|  |  |  |  |
| --- | --- | --- | --- |
| **Function** | **Variable** | **Value** | **Effect** |
| Natural log | a | |a| > 1 | Vertical Stretch |
| Natural log | a | 0 < |a| < 1 | Vertical Compression |
| Natural log | a | a < 0 | Reflect across horizontal |
| Natural log | b | b > 0 | Shift right |
| Natural log | b | b < 0 | Shift left |
| Natural log | c | c > 0 | Shift up |
| Natural log | c | c < 0 | Shift down |
| Exponential | d | |d| > 1 | Vertical Stretch |
| Exponential | d | 0 < |d| < 1 | Vertical Compression |
| Exponential | d | d < 0 | Reflect across horizontal |
| Exponential | f | f > 0 | Shift right |
| Exponential | f | f < 0 | Shift left |
| Exponential | g | g > 0 | Shift up |
| Exponential | g | g < 0 | Shift down |